Virial theorem in astrophysics

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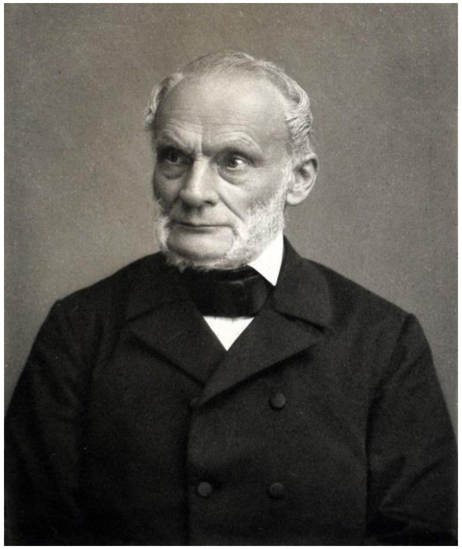
The Virial theorem in astrophysics

- The classical derivation of the Virial Theorem
- The Variational Form of the Virial Theorem
- Some Applications of the Virial Theorem in stellar astrophysics
- Applications of the Virial Theorem in modern physics

The classical derivation of the Virial Theorem

- Earliest clear presentation of the Virial theorem.

Nicolas Léonard Sadi Carnot



Rudolf Clausius

- Average kinetic energy is equal to 1/2 the average potential energy.
- Systems can be described by solving the force equations representing the system.
- The Virial theorem generally deals with scalar quantities.

The classical derivation of the Virial Theorem

- Claussius version for derivation of the theorem.
- Fundamental structural equations of stellar astrophysics.
- Basic conservation laws.
- For the gravitational potential, we arrive at a statement known as Lagrange's identity:

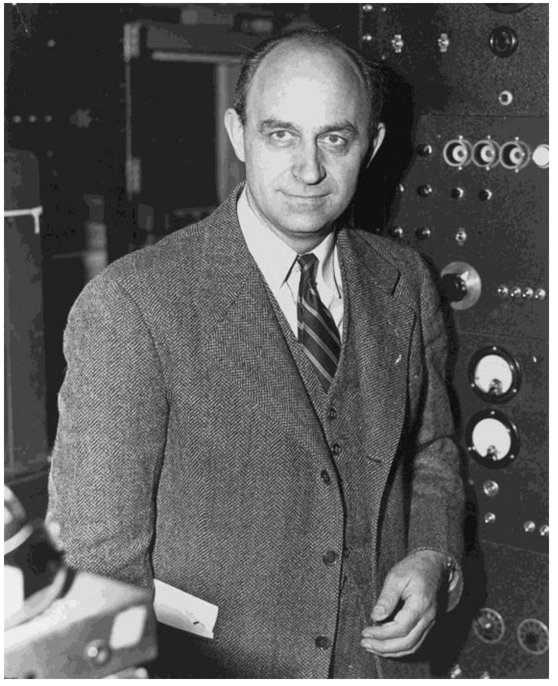
$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2T + \Omega$$

- System is in a steady state.
- Virial theorem:

$$\boxed{2\bar{T} + \bar{\Omega} = 0}$$

The Variational Form of the Virial Theorem

- The variational approach yields differential equations which describe parameter relationships for a system disturbed from an initial state.
- Ledoux determed the pulsational period of the star.
- Chandrasekhar and Fermi investigated the effects of a magnetic field on the pulsation of a star.



Enrico Fermi

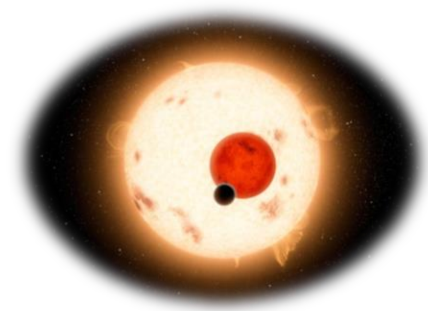
Radial Pulsations for Self-Gravitating Systems: Stars

- Frequency of radial pulsations in a gas sphere.
- The motion is simply periodic.
- The pulsation increases radially outwards in a linear manner.
- Equation for pulsational period takes simple form:

$$T = \left(\sqrt{\frac{4\pi}{3G}}\right)\rho^{-\frac{1}{2}}$$

- This law has been found to be experimentally correct in the case of the Classical Cepheids:

$$0.3d < T < 90d$$



The Influence of Magnetic and Rotational Energy upon a Pulsating System

- Frequency of pulsation of that system.
- All deviations from equilibrium shall be small.
- Radiation pressure will be considered to be negligible.

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2T_{\kappa} + \Omega + \mathcal{M}$$

- Periodic form for the pulsation and linearly increasing amplitude.
- The pulsation period will be positive only if: $|\Omega_0| > \mathcal{M}_0$
- The introduction of magnetic fields only serves to lengthen the period of pulsation.
- The addition of rotational energy will tend to reduce it.

Some Applications of the Virial Theorem: Pulsational Stability of White Dwarfs

- White dwarf become unstable when its radius shrank to about 246 Schwarzschild radii or 1000km.
- Spherically symmetric equation of motion.
- Again, we form LI and its variation.
- We can write eq for stability of white dwarfs as:

$$(R_0/R_S) > 228(M_0/M)^{4/9} \cong 200$$

The Influence of Rotation and Magnetic Fields on White Dwarf Gravitational Instability

- Strong gravitational field.
- The case for rigid rotation.
- Stability conditions:

$$\left(\frac{R_0}{R_s}\right)^3 + \left(\frac{R_0}{R_s}\right)\frac{4.4 \cdot 10^4 w^2}{(1-\mathcal{H}^2)} - \frac{9.3 \cdot 10^{-6}}{(1-\mathcal{H}^2)} > 0$$

- For the field to make any noticeable difference it will have to be truly large.
- Neither magnetic fields nor rotation can significantly alter the fact that a white dwarf will become unstable at or about 1000km.

Stability of a Neutron Star

- The electrons achieve relativistic velocities.
- Let us ignore the effects of general relativity and just consider the special relativistic Virial theorem.
- Stability conditions:

$$\left(\frac{R_0}{R_s}\right) > 4.3$$

- R_o for a neutron star would have to be greater than about 12km
- The effects of magnetic fields and rotation are qualitatively the same for neutron stars as for white dwarfs.

Applications of the Virial Theorem in modern physics: Temperature of the interior of a star

- Planck Quantum Theory of Radiation.
- Most effectively using the Virial Theorem.
- Gravitational potential energy: $V = -\frac{3GM_s^2}{5R}$
- Single atom moving in the interior of the star has a mean kinetic energy <Ke> given by energy equipartition: $\langle K_e \rangle = \frac{3}{2} k \langle T_s \rangle$,
- Virial theorem gives us:

$$\langle T_s \rangle \approx \frac{GM_s^2}{5kNR} = \frac{GM_s^2 m}{5kR}$$

Applications of the Virial Theorem in modern physics: The dark matter hypothesis

- A cluster of galaxies.
- The main portion of luminous and observable matter in a thin disk.
- Gravitational force = Centripetal force.
- The velocity of the star depends on the mass of the galaxy.
- Applying the Virial Theorem to each star:

$$\sigma_r^2 \approx \frac{GM_{virial}}{5R}$$

- Electro-magnetic spectrum analysis.
- F. Zwicky found that there was about 100 times more mass than the expected one given by the spectrum analysis.

Bullet Cluster

- A clash of two galaxies
- The clash caused a strong separation of the individual components
- The homogeneously distributed gas clouds interacted significantly

Bullet Cluster, blue: gravitational potential, red: x-ray emitting gas

- The maximum of the gravitational potential is not the maximum of the visible matter density.
- Dark matter!

Conclusion

- Average kinetic energy is equal to 1/2 the average potential energy.
- Frequency of radial pulsations in a gas sphere. (pulsational period)
- The introduction of magnetic fields only serves to lengthen the period of pulsation.
- The addition of rotational energy will tend to reduce it.
- Equations for stability of white dwarfs and neutron stars.
- Temperature of the interior of a star.
- The dark matter hypothesis.

Thank you for your attention

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